in Alaska. Juneau is to become a regular Weather Bureau station and climatological section center. Not only will the climate of Alaska become more fully known but also it is thought that the general weather and storm forecasts for the United States will be helped.

Another $10,000 is to be used in extending the river and flood and the frost-warning services.

The Weather Bureau has recently announced a new civil service examination designed principally for college graduates who are competent to carry on scientific investigation. The initial salary is $1,260 a year.

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SPECIAL ARTICLES

EXPERIMENTS WITH THE FOUCAULT PENDULUM

1. *Introductory.*—In view of the relatively large angular velocity of the earth, it should be possible to exhibit this rotation by aid of the Foucault pendulum in a few minutes, and this in such a way that reasonably accurate quantitative results may appear. As the pendulum partakes of the rotation of the earth it is not feasible to attach mirrors to the bob, even if this were useful. It is equally clear that the combination of a horizontal pendulum and a Foucault pendulum at its end, or of a large pivoted balance beam with two identical pendulums at its ends will lead to no solution of the problem. In the following note I shall give the results of an optic and of an electric method which I recently had occasion to test and which may interest the reader. A few remarks will also be made on an earth inductor pendulum.

2. *Apparatus.* The question is obviously solved if the swing of the pendulum is regarded with a distant telescope with an ocular micrometer, sighting in the plane of vibration. The equivalent objective result may be obtained if as in Fig. 1, a lens L (not too strong) is placed near the pendulum. The string at rest C is to be at the conjugate focal distance u to the distance v of the screen S from the lens. The string must be strongly illuminated by a Nernst burner N, or sunlight, or the like, and the arc of vibration ab or $cd = D$ must not be so large as to seriously throw the image m of the string at S out of focus. A lens of focal distance of about 60 cm., for a swing D (double amplitude) not larger than 30 cm., does very well. If S is about at 6 meters u will be somewhat short of 70 cm. The pendulum bob should obviously be heavy (3–6 kg.) and the string long (4–5 meters) so that vibration may be slow (period 4 seconds or more), air currents ineffective and observation at S easy.

The vibration is started with the arc ab in the direction of the optical center of L, or otherwise the lens is so placed. In this case the image of the string is stationary at m on the screen. Of course lateral vibration and rotation of the bob around the string as an axis must be scrupulously avoided. This is easily done by letting the bob fall from a lateral hitching cord with one hand after all vibration has been checked by the loose fingers of the other hand, and the image is at m.

The image m soon begins to vibrate right and left more or more fully on the screen S and after the earth has rotated over the angle θ, the point c is replaced by the elongations dd' and the point m has expanded into the elongations at a distance x apart. With a swing of $D = 36$ cm. originally, the distance x increases to nearly 5 cm. in 5 minutes, or about 1 cm. per minute with the dimension of pendulum and lens given above. The rate falls off because the arc D diminishes.

3. *Equation.*—Fig. 1 shows that if θ is the angle of rotation, for the distance x between the elongations at the screen S and the double swing of pendulum $cd = D$, and if the constant $k = u/v$, approximately

$$(1) \qquad \theta' = \frac{u - D/2}{v}\frac{x}{D} = (k - D/2v)x/D,$$

remembering that the angles θ' at c remain small and are initially nearly the same as θ at the center. Furthermore with the same approximation

$$(2) \qquad \theta = \theta'(1 + D/2u) = k(1 - D^2/4u^2)(x/D).$$

Hence after reduction if the rates per hour be dotted

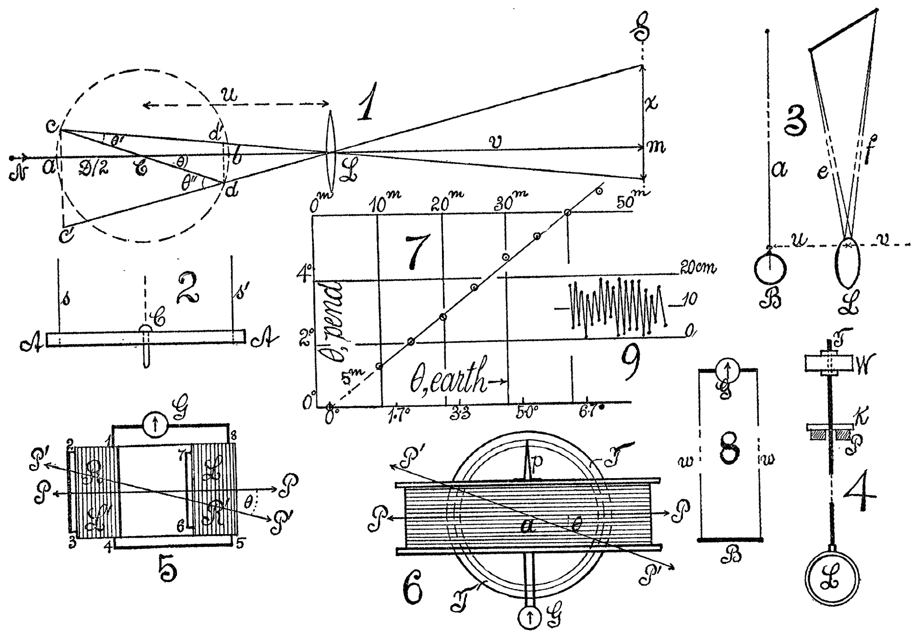

FIG. 1–9

(3) $\dot{\theta} = k\left\{\left(1-\frac{D^2}{4u^2}\right)\frac{\dot{x}}{D} - \left(1+\frac{D^2}{4u^2}\right)\frac{x\dot{D}}{D^2}\right\}.$

The term in $\dot{D}$ increases rapidly.

Another approximation which is perhaps equally good is much simpler. This consists in regarding the angle θ as the mean of θ' and θ'' in Fig. 1; whence

$$\theta = kx/D. \qquad (4)$$

Here k is constant and

$$\dot{\theta} = k\frac{x}{D}\left(\frac{\dot{x}}{x} - \frac{\dot{D}}{D}\right). \qquad (5)$$

Values so obtained are usually too large and increase in the lapse of time, whereas values of equation (3) decrease.

4. *Experimental Reduction.*—The difficulty in correcting the results in x by the equation given makes it desirable to standardize the apparatus directly. This may be easily done by aid of a horizontal arm AA, Fig. 2, carrying two fine vertical wires s, s' at a distance $D = 25$ cm. (average arc above) apart, rotating around an axis C over a graduated circle (not shown). The axis C is to coincide with the string of the pendulum, Fig. 1, and the lens L to correspond as before to the conjugate focal distances u and v. In this way the angle θ is directly determined in terms of x at the screen S, apart from all optic considerations. For the dimensions given, s and s' are adequately focused at S. The data show that within an angle less than 10°, θ may be regarded as proportional to x/D.

This method reproduces the actual conditions under which pendulum observations are made and there seems to be no reason for calling the result in question.

Another method consists in finding the magnification by placing a millimeter scale at C, Fig. 1, and measuring its image x at S.

Both these methods have an advantage, as they admit of reducing the individual x, D values to θ values, without requiring differential coefficients.

5. *Observations.*—The first experiments were made with an ordinary plumb bob somewhat lighter than a pound, swinging from a silk thread over 4 meters long. Fair results were obtained but the light bob is not always trustworthy. An example which must suffice here is given in Table I. for an unnecessarily heavy

pendulum bob (4 kg.), swung from a thin brass wire (diameter .7 mm.). Measurements were continued throughout 35 minutes.

TABLE I

Pendulum Observation

Stationary lens, focal distance 60 cm.; diameter 15 cm. Bob 4 kg. on brass wire. (1) Experimentally $\theta_1 = 1°$ equivalent to $x = 3.8$ cm. for $D = 25$ cm. (2) Experimentally $\frac{1}{2}D\theta_2 = 1$ cm. equivalent to $x = 9$ cm. for $D = 25$ cm.; $K_0 = \frac{u}{v} = \frac{68}{603} = .1144$ $\dot{\theta} = 15° \times \sin \phi = 10.01\ \dot{\theta}^{0/h}$ in Providence.

Time, Min.	x, Cm.	D, Cm.	$\dot{\theta}_1$, °/h	$\dot{\theta}_2$, °/h	$\dot{x}/6$ Cm./h	Mean x, Cm.	$\dot{D}/6$ Cm./h	Mean D, Cm.	$\dot{\theta}' + \dot{\theta}'' =$	$\dot{\theta}$
0	1.0	36	—	—						
5	5.8	34	11.2	11.1	8.4	5.2	3.5	3.4	9.2°+.6°=	9.8°
10	9.4	32	10.2	10.1	7.0	9.3	3.5	3.2	8.2 + 1.3=	9.5°
15	12.8	30	10.2	10.2						
20	15.3	29	9.7	9.7						
25	17.8	27	9.7	9.6						
30	20.1	27	9.3	9.3						
35	22.0	25	9.3	9.3						

$\dot{\theta}_1$ from direct measurement of θ and x.

$\dot{\theta}_2$ from direct measurement of magnification: z at u and x at v. $\dot{\theta}' + \dot{\theta}''$ by the general equation (3) § 3. By equation (5), $\dot{\theta} = 10.4°$ and 9.9°, respectively.

The results for $\dot{\theta}_1$ (computed from the direct evaluation of θ, §4) and for $\dot{\theta}_2$ (computed from direct measurement of magnification §4) are practically identical. These data for $\dot{\theta}$ decrease in the lapse of time, definitely. In part this may be ascribed to an insufficiently accurate estimate of the arc D of the pendulum, for which a value derived from the logarithmic decrement might with advantage have been substituted. The high initial value is in part to be associated with an incorrect initial zero. But it is also probable that some secondary disturbance is developing and superimposed on the data for the earth's rotation.

The value of $\dot{\theta}$ found from the equation (4) is given in the second part of Table I. with the mean data used, for the first four observations taken in pairs. It is of about the same order as the others and also gives promise of decreasing.

6. *The Vibrating Lens Pendulum.*—To increase the magnification indefinitely, *i. e.*, to exhibit the rotation θ in shorter time, it will be necessary to use the lens L, Fig. 3, as the bob of a pendulum, swung doubly bifilarly, or in some similar manner, but in such a way as to have the same period as the Foucault pendulum, B. As the bifilar suspension is still liable to vibrate laterally it is unsuitable for this and other reasons. It was therefore replaced by a massive compound pendulum LT, Fig. 4, about a meter long, weighted above with 1.5 or 2 kilograms to secure as long period as that of the Foucault pendulum (4 seconds). The steel knife edge at K should rest on a horizontal flat brass fork P, as it will be necessary to rotate the pendulum slightly around its longitudinal axis LT in the adjustments. The weights W are between screw bolts to regulate the period. The lens L used was an ordinary photographic bullseye lens, 10 cm. in diameter, quite thick and with a focal distance of about 10 cm. The magnification was between 62 and 65.

As the distance between B and L, Figs. 3 and 4, is but 10 cm. the weights W interfere with the string for large arcs of vibration, D. This would have to be modified in a lecture apparatus, for instance by doubling the lens (condenser doublet) or by forking the weights. Furthermore the vibrations of L die down more rapidly than those of B. Since however the pendulum L is weighted above, there is no difficulty in accelerating the lens L cautiously with the fingers when necessary before observation.

In adjusting the apparatus, B must first be quite at rest. The pendulum L is then started, and if the image of the wire of B vibrates on the screen, the lens L is to be rotated on its longitudinal axis, by successive trials, until the image is stationary. Hence the arc traced by the optical center of L passes through the wire of the Foucault pendulum. B is now to be deflected as above and held until the image of the wire is still fixed in the same place, after which B is released with the two pendulums in step. These operations succeed much easier than would be expected.

The results obtained with this apparatus are essentially exhibitional. Thus far 30-minute intervals if x is the mean arc on the screen s and D the mean swing from which, for the

Time	$\dot{x}/2$	Mean x	D
30 min.	55 cm.	30 cm.	11 cm.
30 min.	60 cm.	50 cm.	10 cm.

magnification 60, the angles $\dot{\theta} = 9.6°$ and $11.5°$ roughly follow. As the x is equivalent to 2 cm. per minute for the swing $D = 10$ cm. this implies 5 cm. per minute for the usual swing of 25 cm. The experiment is therefore striking, but the necessary interferences make it untrustworthy for absolute values of $\dot{\theta}$. Under all circumstances care must be taken that the lens vibrates without displacing the image of the pendulum wire (at rest), both at the beginning and at the end of the experiment.

7. *Electrical Methods*.—The preceding methods are essentially exhibitional, since the measurements are made from images out of focus. It seems possible however that by the use of the following electrical device a method of precision might eventually be evolved, though this is not attempted in the present paper. In all these cases the pendulum bob is a massive cylindrical magnet weighing .8 kg., 20 cm. long and 25 cm. in diameter, with its axis in the prolongation of the string and its north pole downward. The bob is to be additionally and symmetrically weighted. Its arc of vibration is along PP in Figs. 5 and 6. In case of the former four identical coils R, R', L', L, on a wooden core about 5 cm. square were placed symmetrically to the line PP and just below the magnetic bob. The currents induced in R and R' are guided to counteract those in L and L' in an otherwise continuous circuit, so that the galvanometer at G indicates the differential current. If the system $RR'LL'$ is symmetrical to PP the current in G is zero. If PP deviate to PP' the current in RR' will be in excess. The zero may then be restored by rotating $RR'LL'$ until PP and $P'P'$ coincide. This and other methods were tested, but a more elegant design is given in Fig. 6 in which CC is a long coil with strands of wire wound in the direction of the original arc of vibration PP. The coil which I used was about 30 cm. long wound on a square wooden core 5×5 sq. cm. in cross section, with 6 layers of 34 turns each of copper wire .8 mm. in diameter. The terminals of the coil lead to the galvanometer G, an astatic instrument (preferably), with mirror. The coil CC with the pointer p must be capable of revolving around a vertical axis at a, over the fixed graduated circular plate TT for the measurement of the angle θ in standardizing the instrument.

It is obvious that so long as the pendulum vibrates in the plane PP, the induced electromotive force is normal to the strands of wire and the current at G is zero. When the vibration is oblique, along $P'P'$ for instance, there is a component electromotive force along the strands and the current at G increases rapidly with θ. If the period of the needle is about equal to that of the pendulum the arrangement is quite sensitive and an image of a Nernst filament reflected from the mirror of the needle soon oscillates across a distant wall or screen.

To obtain the current zero, the magnetic bob must oscillate strictly in the vertical plane PP. Any cross vibration or elliptic oscillation at once develops marked currents. Moreover in the course of time it is extremely difficult to obviate the development of these cross vibrations. They would arise if the bob rotates around its own axis, since rigorous rotational symmetry is rarely attained. They would also arise in the reaction of induced currents on the magnetic pole.

The following is a typical experiment among many results. A galvanometer with astatic needles was adjusted by aid of three astisizing magnets placed symmetrically below and on the sides of the needle (strengthening the earth's field) until its period was decreased to 4 seconds, nearly identical with that of the pendulum. In view of this relatively strong magnetic field, the needle was practically free from damping resistances. The experiment was very striking, for with an arc of vibration D between 20 cm. and 25 cm., the vibration of the image of a Nernst filament at first $(D = 25)$ increased over 3 cm. per minute.

Table 2 and Fig. 7 is an exhibit of the data obtained when the plane of the pendulum vibration passed through the plane of the coils, x changing from negative to positive values. Unfortunately the undamped needle does not stop vibrating when the intensity of the inductive impulse is reduced to zero; otherwise the rotation of the earth might be directly read off at p, Fig. 6, by rotating the coil on a tangent screw. The reduction factor F in $\theta = Fx$ was measured for 3 arcs: At $D = 24$ cm., $\theta = .054°$, at $D = 14$ cm., $\theta = .087°$ and at $D = 8.7$ cm., $\theta = .111°$, corresponded respectively to $x = 1$ cm.

TABLE II

Electrical Pendulum Registry

$\theta = Fx - .5°$ cylindrical magnetic bob, length 20 cm., diam. 2.5 cm., mass 800 grams. Long square coil, 6 layers, 34 turns each, 30 cm. long, 5 cm. broad, 5 cm. high within. Bob and astatic needle of galvanometer with synchronized period of 4 sec.

t, Min.	D, Cm.	$F \times 10^3$	x, Cm.	θ, Deg.	$\dot{\theta}$, °/Hour
0	22	60	—19	—	—
5	19	70	(7)	(.00)	—
10	16	80	16	.78	9.4
15	14	85	24	1.54	9.2
20	13	90	31	2.29	9.2
25	11	100	37	3.20	9.6
30	10	105	44	4.12	9.9
35	9	110	48	4.78	9.6
40	8	115	52	5.48	9.4
45	7	125	53	6.13	8.2

For other arcs D the reduction factor F was interpolated. When x is negative, the arcs x are in excess of the electromotive impulses which are decreasing toward zero. When x is positive the arcs are in deficiency of the increasing impulses due to the rotation of the earth. Hence an undamped needle does not come to rest and in Table II. and Fig. 7, $\theta = 0°$ at $t = 2$ min. was interpolated (parenthesis) from the subsequent 8 data. This makes $\theta = Fx - .5°$, beginning with $t = 5$ min. The fluctuations of $\dot{\theta}$ are due to the rough measurement of D and the correspondingly rough value of the reduction factor F and are quite as good as anticipated. Eventually the decrement of x due to decreasing arc D must begin to approach the increments due to the earth's rotation, whereupon x will be stationary. This seems to happen after 45 m, in Table II.

Again if the reduction factor F of x is taken constant throughout, the results show the rapidity with which the θ values fall off even after 10 minutes. Thus it seems that a compound pendulum on knife edges, Fig. 4, with the magnetic bob similarly placed to the coil must be used for standardization.

In other series experiments the reduction from x to θ was made linearly, the constants being a mean approximation from a direct measurement of x and θ. This however is the real difficulty of the method and is far from satisfactory owing to the development of cross vibrations.

In the final results the case of a core of 4 iron plates (each 18 cm. $\times$ 25 cm. $\times$ .044 cm.) placed symmetrically within the coil was tested. In view of the breadth of these plates and the weight of the pendulum there was supposed to be no danger from induction. The sensitiveness (scale at 4 meters) was thus increased to an initial growth of $x = 5$ cm. per minute of earth rotation. It would have been larger if the periods of pendulum and needle had been as nearly the same as before. Here I found roughly $\theta = Fx = (.110 - .0035D)x$ and it was interesting to note that for the last data the term in Dx had passed through a maximum. Hence the increments of x are much reduced. If the logarithmic decrement is used, $\dot{\theta} = 60(a - bD_0c^{t/5})x$ degrees per hour, follows, where a and b are the constants given, $D = 27$, $c = .896$. Greater smoothness is thus obtained, but the real difficulty which resides in the constants a and b is left untouched. Finally one may note that the data with a plate iron core in the coil were apparently as good as those obtained without; for the correction coefficients which indicate the growth of cross vibrations were actually larger (accidentally) in the absence of iron.

8. *Short Pendulum.*—The endeavor was now made to use the same method for a short pendulum. For this purpose the magnetic cylinder was swung on a round glazed fish line.

To secure an adequate suspension the top of the cord was first passed through a snugly fitting hole in a fixed wire draw-plate and then attached to the shaft of a strong fixed horizontal screw, above. On turning the screw the bob could be raised or lowered at pleasure or secured in any position in virtue of the friction of the screw. An old Kohlrausch galvanometer with elliptic coils and a magnetized steel mirror in a copper damper at its center was found very serviceable. By placing the astisizing magnet in different positions with or against the earth's field, the periods could be usefully varied from 1 second to over 6 seconds.

Pendulums $\frac{1}{2}$ to 1 meter in length were first suspended from a single massive rigid standard; thereafter from a gallows between two massive standards, carefully braced. In neither case was I able to eliminate to development of elliptic vibrations, however, resulting either from the action of the induced currents on the magnetic bob (an effect to be anticipated) or from vibrations at the suspension. I did not therefore attempt to carry out measurements, although from the rapid motion, the sensitiveness was very marked, $\theta = .06°$ to $.03°$ per $x = 1$ cm. being easily available. A rotational effect should therefore be observable in 10 sec. The whole experiment is an interesting one, regarded either in its present bearing, or as an illustration of a vibrating system of two degrees of freedom, or of the laws of induction.

9. *The Bifilar Inductor Pendulum.*—Though not immediately connected with the present subject, the following striking experiment uses similar synchronized apparatus. A long (1–2 meters) brass or copper rod or bob, *B*, Fig. 8, is swung horizontally from two thin vertical brass wires *ww* attached at the ends of the rod and to the ceiling, or elsewhere. These thin wires are the terminals of the synchronized galvanometer, *G*, and the brass rod swings parallel to itself, cutting the earth's vertical magnetic field, H_v, normally. The mean horizontal speed, $\dot{y}$, of the rod may be written in terms of the maximum speed, $\dot{y}_0$ (simple harmonic motion) as $\dot{y} = 2\dot{y}_0/\pi$ and if a is the amplitude of the pendulum, T its period, l its length, g the acceleration of gravity, e is the mean electromotive force induced and b the length of brass rod (bob),

$$e = \frac{2abH_v}{\pi \times 10^8}\sqrt{\frac{g}{e}} \text{ volts.}$$

In my pendulum

$a = 20$ cm., $b = 100$ cm., $H_v = .4$, $l = 400$ cm., whence

$$c = 8 \times 10^{-6} \text{ volts nearly.}$$

Thus it should be possible to measure e with a moderately sensitive galvanometer, particularly so if its period is the same as that of the pendulum.

Incidentally one may observe that if a horizontal wire 10 meters long is moved normally through the earth's vertical field with a speed of 2 kilometers per minute, as on a flying machine, the difference of potential at the ends would be over $e = 10^{-2}$ volts. The latter would have to be measured electrostatically, however, with an artificial earth like a large insulated condenser. If this can be done, it would suggest a method of registering the speed of the machine.

A number of experiments were made with the above pendulum ($T = 4$ seconds) and the synchronized Kohlrausch galvanometer, of which Fig. 9 gives an example. The needle of the galvanometer was not at rest, owing to the proximity of trolley wires and the astasized *simple* needle. Hence the fluctuations at the two elongations. But apart from this, the result is about $x = 7$ cm. between elongations per meter of length of the bob of the bifilar pendulum and a double amplitude of the latter of about $D = 40$ cm. (screen at 4 meters). A shorter pendulum, an astatic needle and an external magnet strengthening the earth's field at the galvanometer, would give smooth results. D could be much increased, etc. It is also obvious that a long rectangular coil similar to the bifilar and on knife edges could be used to multiply the effect of the single bifilar circuit.

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